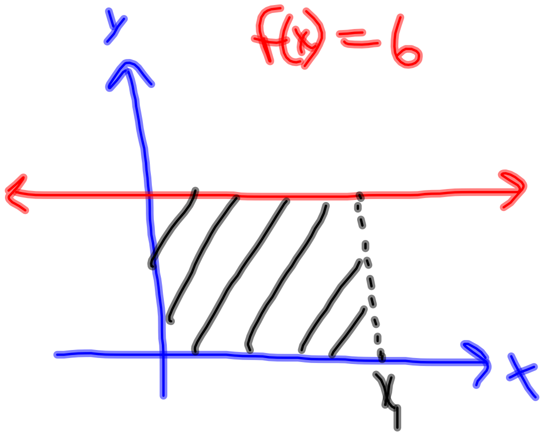
Finding the Area under a curve (that actually curves)



$A = 6(x_1)$

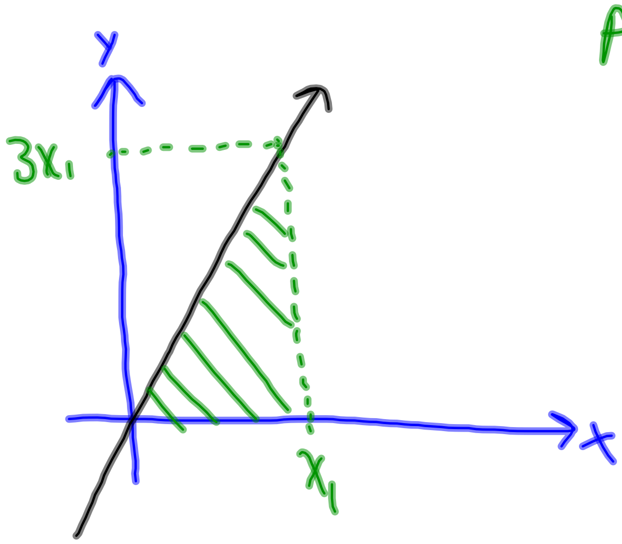
note that the area is a function of x

$A(x) = 6x$

$A(3) = 18$

$A(7) = 42$

$f(x) = 3x$



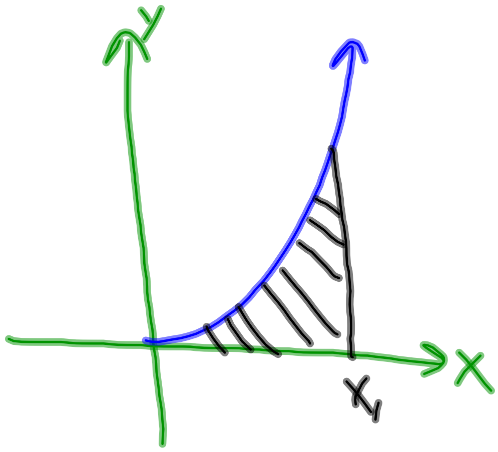
$A = \frac{1}{2} x_1 3x_1 = \frac{3}{2} x_1^2$

$A(x) = \frac{3}{2} x^2$

$A(2) = 6$

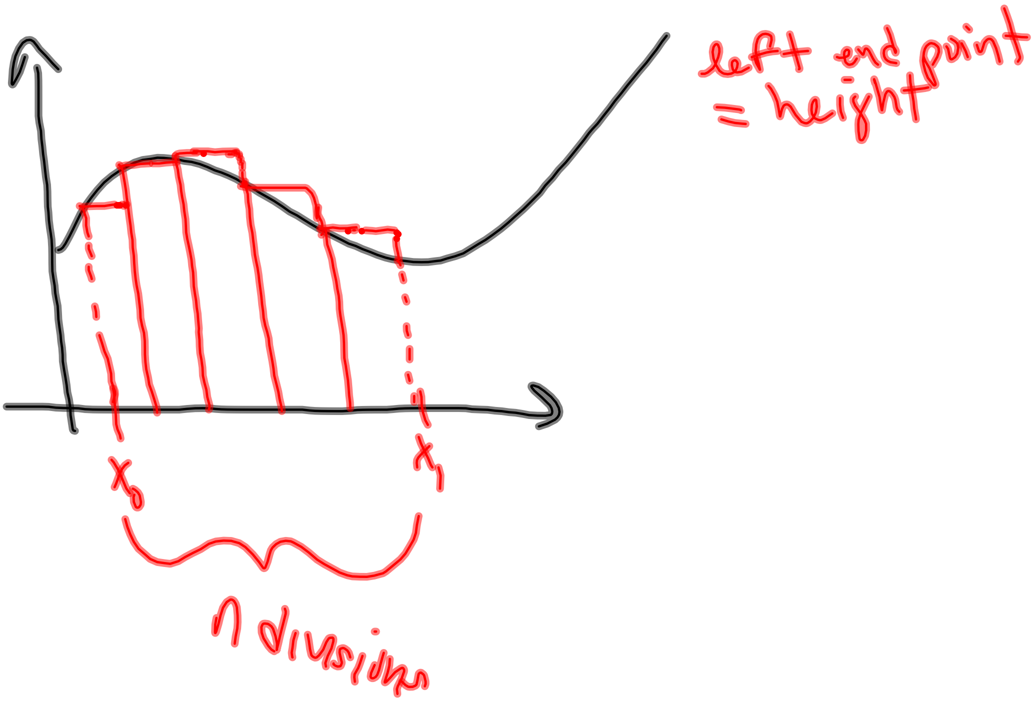
$A(7) = \frac{147}{2}$

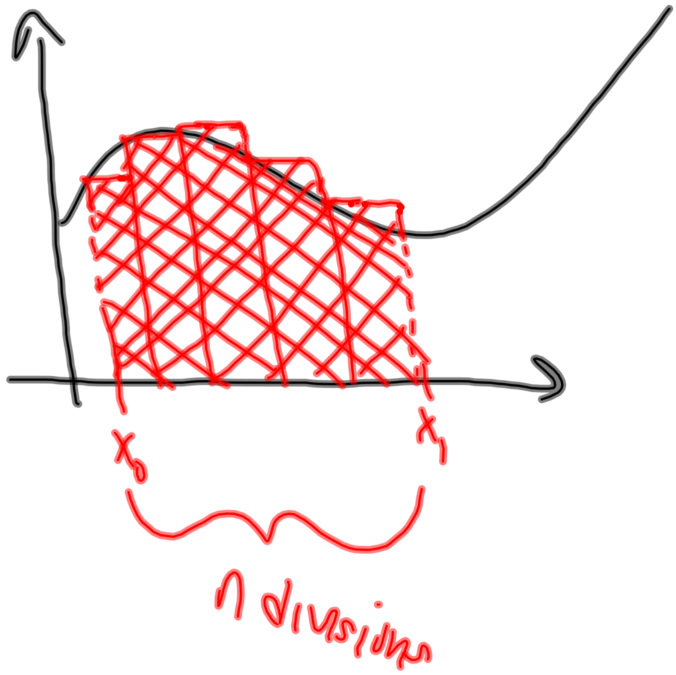
$f(x) = x^2$



$A = ??$

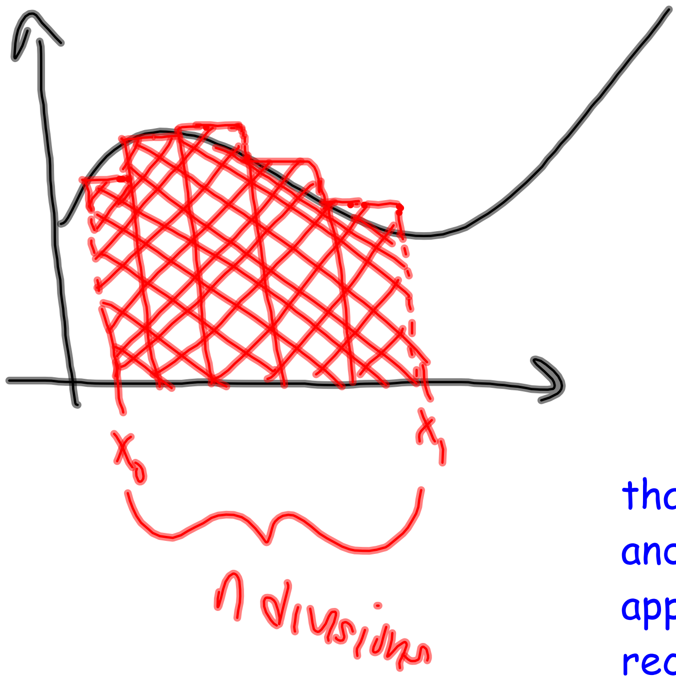
rectangle method pg 380





left end point = height

close but not exact :(



What happens to the little error parts as n gets large without bound?

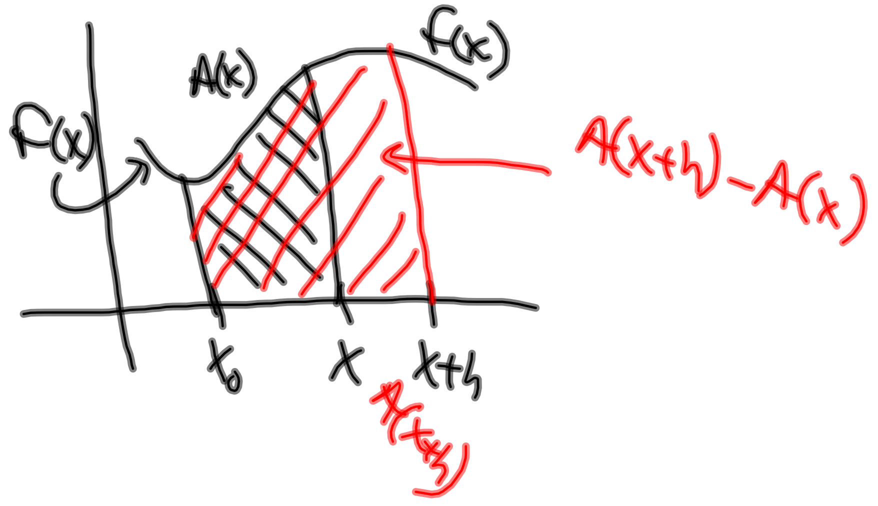
thats right :) they approach 0, and if we take the limit as n approaches inf. the sum of the rectangle's area approaches the EXACT area under the curve :)

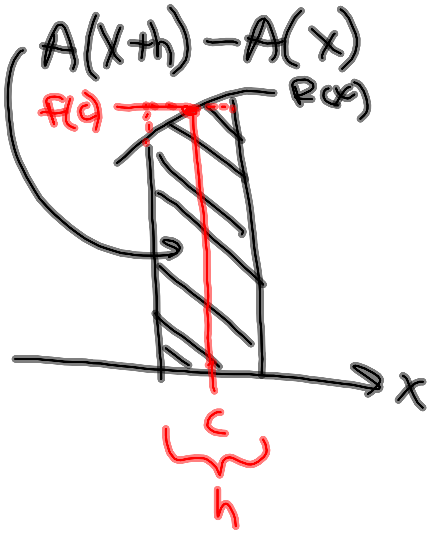
$A(x)$ is area as a function of x

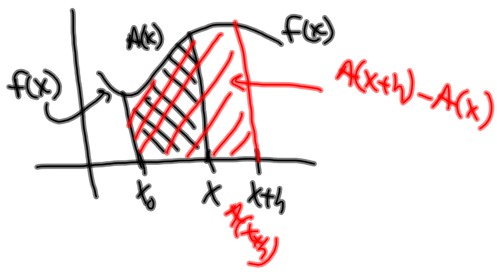
for any function

$$A'(x) = \lim_{h \to 0} \frac{A(x+h) - A(x)}{h}$$

numerator is just diff of 2 areas







$$f(c)\cdot h \approx A(x+h)-A(x)$$

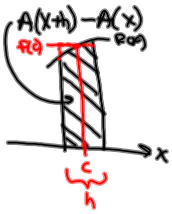
Yes? as $h \to 0$ =

f(c)*h = area of rectangle

A(x+h)-A(x) = area under curve

$$A'(x) = \lim_{h \to 0} \frac{A(x+h) - A(x)}{h} = \lim_{h \to 0} \frac{f(c) \cdot h}{h}$$

$$= \lim_{h \to 0} f(c)$$



as $h \to 0$

$c \to x$

$$= f(x)$$

$A'(x) = f(x)$

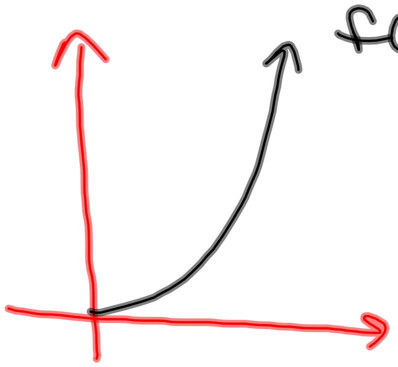
b.f.h.d.

This result is what the rest of the course is all about. It is that simple :) ... really.
(again, notice I said "simple", not "easy")

$$A'(x) = f(x)$$

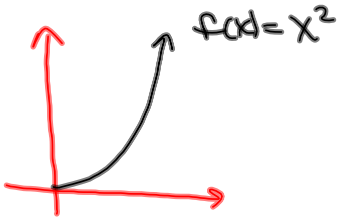
Find some function A(x) whos derivative is f(x)

$f(x) = x^2$



how about $A(x) = \frac{1}{3}x^3$?

so the area under x^2 is $\frac{1}{3}x^3$



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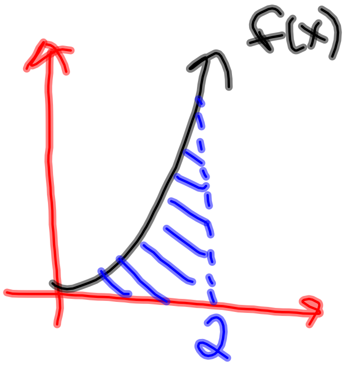
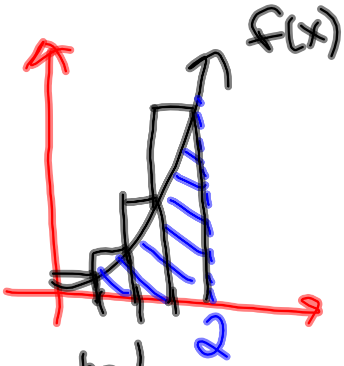


$\frac{1}{3}x^3$

$\frac{1}{3}(2)^3 = \frac{8}{3}$

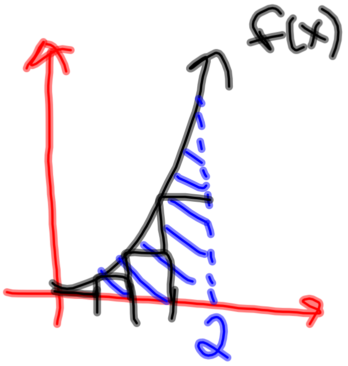
2.67



$h = \frac{1}{2}$

$$A = f\left(\tfrac{1}{2}\right)\cdot\tfrac{1}{2} + f(1)\cdot\tfrac{1}{2} + f\left(\tfrac{3}{2}\right)\cdot\tfrac{1}{2} + f(2)\cdot\tfrac{1}{2}$$

$$A = \frac{1}{8} + \frac{4}{8} + \frac{9}{8} + \frac{16}{8} = \frac{34}{8} = 4.25$$



$A = f(0)\cdot\frac{1}{2} + f(\frac{1}{2})\cdot\frac{1}{2} + f(1)\cdot\frac{1}{2} + f(\frac{3}{2})\cdot\frac{1}{2}$

$A = 0 + \frac{1}{8} + \frac{4}{8} + \frac{9}{8} = \frac{14}{8} = 1.75$

$\frac{4.25 + 1.75}{2} = 3.0 \approx 2.67$

$n = 4$

What about $n = 400$?

So, the function we found, which we will call and "antiderivative" gave us a good value for the area under the curve. Now the problem becomes, how to find these antiderivatives.

The Indefinite Integral

the process of finding antiderivatives is called "Integration"

$f(x) = 6$ antider $A(x) = 6x$

$f(x) = 3x$ antideriv $A(x) = \frac{3}{2}x^2$

$f(x) = x^2$ antideriv $A(x) = \frac{1}{3}x^3$

are these the unique antiderivatives?

$f(x) = 6$ antidv $A(x) = 6x + 2$

$f(x) = 3x$ antidvo $A(x) = \frac{3}{2}x^2 + 7.6$

$f(x) = x^2$ antidvu $A(x) = \frac{1}{3}x^3 + \pi$

for $f(x)$ we have $A(x) + C$

"constant of integration"

$$\frac{d\left(A(x)+C\right)}{dx} = f(x)$$

Notation

$$\int f(x)\,dx = A(x) + C$$

formulas pg 384

look how much we know already !!

$$\frac{d(\sin x)}{dx} = \cos x \qquad \int \cos x\,dx = \sin x + C$$

$$\frac{d(\ln x)}{dx} = \frac{1}{x} \qquad \int \frac{1}{x}\,dx = \ln x + C$$

integrate

$f(x) = \frac{1}{x} = x^{-1}$

$A(x) = x^0 = 1$?

Nope

$A'(x) \neq \frac{1}{x}$

so $\frac{d(\ln x)}{dx} = \frac{1}{x}$ is way important

properties pg 385
exs 3,4,5

$$\int c f(x) = c \int f(x)$$

$$\int f(x) \pm g(x) = \int f(x) \pm \int g(x)$$

ex 3b) $\int (x + x^2)dx = \int x\,dx + \int x^2 dx$

$$= \frac{x^2}{2} + C + \frac{1}{3}x^3 + C$$

$$= \frac{x^2}{2} + \frac{x^3}{3} + C$$

ex5) $\int \frac{\cos x}{\sin^2 x}\,dx = \int \frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}\,dx = \int \cot x \csc x\,dx$

$$\frac{d(\csc x)}{dx} = \csc x \cot x$$

$$= \csc x + C$$

$$\int \frac{t^2 - 2t^4}{t^4}\,dt = \int \left(\frac{1}{t^2} - 2\right)dt \qquad \frac{d(t^{-1})}{dt} = -t^{-2}$$

$$= -\frac{1}{t} - 2t + C$$

integral curves pg 387
differential equations p388 (this is an entire course by itself)

$$\frac{dy}{dx} = x^2$$

$$\int \frac{dy}{dx} = \int x^2 dx$$

$$y = \frac{x^3}{3} + c$$

$$\frac{dy}{dx} = \cos x$$

$y(0) = 1$
initial condition

$$y = \sin x + C$$

$$1 = \sin(0) + c$$

$$1 = c$$

$$y = \sin x + 1$$

HW:
page 382 #4
page 389 #3,5,7,17,19,21,29,39b,43