Substitution with Definite Integrals

remember substitution with
indefinite integrals? (of course you do!)

$$\int (x^2+1)^3 dx \qquad \text{let } u = (x^2+1) \quad du = 2x\,dx$$

$$= \frac{1}{2}\int (x^2+1)^3 2x\,dx$$

$$= \frac{1}{2}\int u^3 du = \frac{1}{2}\frac{u^4}{4} = \frac{1}{8}(x^2+1)^4 + C$$

Jul 25-7:19 AM

Method 1: $\int_0^2 (x^2+1)^3 dx$ let $u = x^2+1$ $du = 2x$

$\frac{1}{2}\int_{x=0}^{2} u^3 du = \frac{1}{2}\frac{u^4}{4} \cdots \frac{1}{2}\frac{(x^2+1)^4}{4}\Big|_{x=0}^{2}$

$\frac{1}{8}\left(5^4-1\right) = \frac{624}{8} = 78$

Method 2:

Same up to $\frac{1}{2}\int u^3 du$ $u = x^2+1$ $x=0 \Rightarrow u=1$

$x=2 \Rightarrow u=5$

$\frac{1}{2}\int_{u=1}^{5} u^3 du = \frac{1}{2}\frac{u^4}{4}\Big|_1^5 = \frac{1}{8}\left(5^4-1\right) = 78$

ex2 $\displaystyle\int_0^{3/4} \frac{dx}{1-x}$

let $u = (1-x)$ $\quad du = -dx$

$x = \frac{3}{4} \Rightarrow u = \frac{1}{4}$

$x = 0 \Rightarrow u = 1$

$$-\int_1^{\frac{1}{4}} \frac{du}{u} = \int_{\frac{1}{4}}^{1} \frac{du}{u} = \ln|u| \Big/_{\frac{1}{4}}^{1} = \ln 1 - \ln\tfrac{1}{4} = \underline{\ln 4}$$

$0 - \ln\frac{1}{4}$

$0 + \ln\left(\frac{1}{4}\right)^{-1} = \underline{\ln 4}$

$0 - (\ln 1 - \ln 4)$

$0 - 0 + \underline{\ln 4}$

ex 3 pg 443

$$\frac{1}{5}\int_0^5 (70-30e^{-.5t})\,dt$$

$$=\frac{1}{5}\int_0^5 70\,dt - 6\int_0^5 e^{-.5t}\,dt$$

let $u=-.5t$

$du=-.5\,dt$

$x=0 \Rightarrow u=0$

$x=5 \Rightarrow u=-2.5$

$$-2\int_{u=0}^{-2.5} e^u\,du$$

$$=-2e^u\Big|_0^{-2.5}$$

$$=-2(e^{-2.5}-e^0)$$

$$-2e^{-\frac{5}{2}}+2$$

$$\frac{1}{5}70t\Big|_0^5 \qquad -6\left(-2e^{-\frac{5}{2}}+2\right)$$

$$\frac{1}{5}(350)-0$$

$$70 \qquad +\frac{12}{e^{5/2}}-12$$

$$\frac{12}{e^{5/2}}+58=58.99$$

2,14,20,28,45

2) a) $u=2x-1$ $x=0 \Rightarrow u=-1$ $du=2x\,dx$

$x=1 \Rightarrow u=1$

$\frac{1}{2}\int_{-1}^{1} e^u\,du$

b) $u=\ln x$ $x=e \Rightarrow u=1$ $du=\frac{dx}{x}$

$x=e^2 \Rightarrow u=2$

$\int_{1}^{2} u\,du$

2c) $u = \tan x$ $x = 0 \Rightarrow u = 0$ $du = \sec^2 x\, dx$

$x = \frac{\pi}{4} \Rightarrow u = 1$

$$\int_0^1 u^2\, du$$

2d) $u = x^2 + 3$ $x = 0 \Rightarrow u = 3$ $du = 2x\, dx$

$u - 3 = x^2$ $x = 1 \Rightarrow u = 4$

$x^3\sqrt{x^2+3}\,dx = x^2\left(\sqrt{x^2+3}\right)(x\,dx)$

$$\frac{1}{2}\int_3^4 (u-3)\sqrt{u}\, du$$

Jul 25-10:49 AM

14) $\int_0^2 x\sqrt{16-x^4}\,dx$ let $u=x^2$ $du=2x\,dx$

$\frac{1}{2}\int_0^2 \sqrt{16-(x^2)^2}\,2x\,dx$

$x=0 \Rightarrow u=0$

$x=2 \Rightarrow u=4$

$\frac{1}{2}\int_0^4 \sqrt{16-u^2}\,du$

$y=\sqrt{16-u^2}$

$y^2=16-u^2$

$y^2+u^2=16$ circle $C(0,0)$ $r=4$

$=\frac{1}{2}(4\pi)=2\pi$

$A=\frac{1}{4}\pi(16)$

Integration and Logarithmic functions

(you feel smarter just reading that out loud!)

fig 7.9.1 p 446
def 7.9.1 pg 447
fig 7.9.2 pg 447
pg 448 7.9.2

form 4 pg 449
example 2

HW:

pg 444 1,5,9,15,21,27,31,47

pg 451 3a,3b,7a,7c,11