Definition of the derivative

(drum roll, please...)

this sounds monumental, even transcendant, but it is soooo simple (notice I didn't say "easy")

The "derivative" of a function is the slope of the tangent to the curve at any point (so the derivative is a function itself) pg 172 bottom left, error in table

change 1.00 to 2.00

read Morris Kline on pg 171

p173 defn 3.1.3 average rate of change

$[x_0, x_1]$

$$m_{sec} = \frac{\Delta y}{\Delta x} = \frac{f(x_1)-f(x_0)}{x_1 - x_0} = \text{avg rate of change}$$

p174 defn 3.1.4 instantaneous rate of change

$$\lim_{x_1 \to x_0} \frac{f(x_1)-f(x_0)}{x_1 - x_0} = m_{tan} = \text{inst. r.o.c.}$$

example 1
p174

a) $y = x^2 + 1 \quad [3, 5]$

$$m_{sec} = \frac{f(5) - f(3)}{5-3} = \frac{26-10}{2} = 8$$

b) $x_0 = -4$

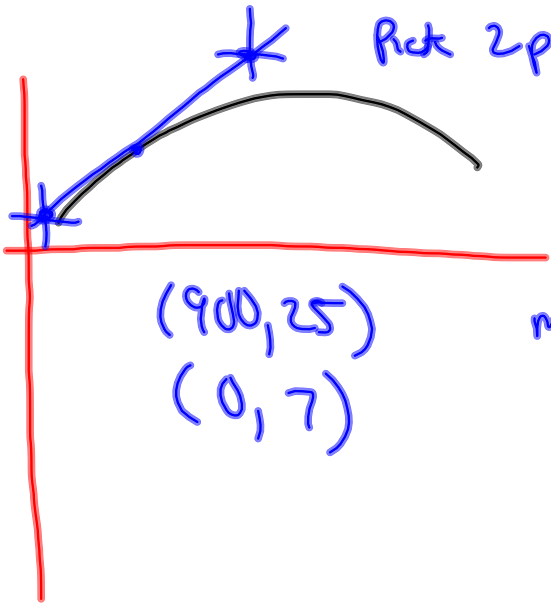
$$\lim_{x_1 \to -4} \frac{f(x_1) - f(-4)}{x_1 - (-4)} = \lim_{x_1 \to -4} \frac{(x_1^2+1) - 17}{x_1 + 4}$$

$$\lim_{x_1 \to -4} \frac{x_1^2 - 16}{x_1 + 4} = \lim_{x_1 \to -4} \frac{\cancel{(x_1+4)}(x_1-4)}{\cancel{(x_1+4)}} = \lim_{x_1 \to -4} (x_1 - 4)$$

$$= -8$$

so, the slope of the tangent to the curve at x=-4 is -8

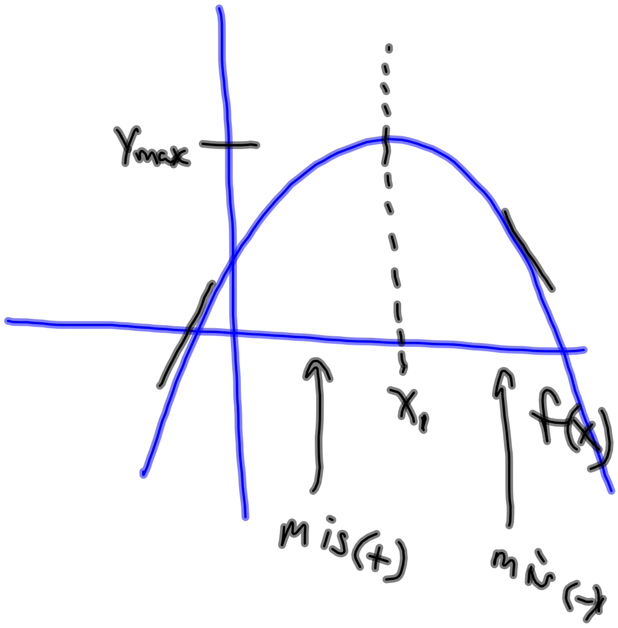
example 2 (end)
p175



Pick 2 points

(900, 25)
(0, 7)

$$m_{tan} = \frac{25-7}{900-0} = \frac{18}{900}$$



m_{tan} at x_1 ??

$= 0$

(horizontal)

defn 3.2.2 pg 179

let $x_1 - x_0 = h$

call $f'(x)$ "the derivative"

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

pg 179 exs 2,3,4

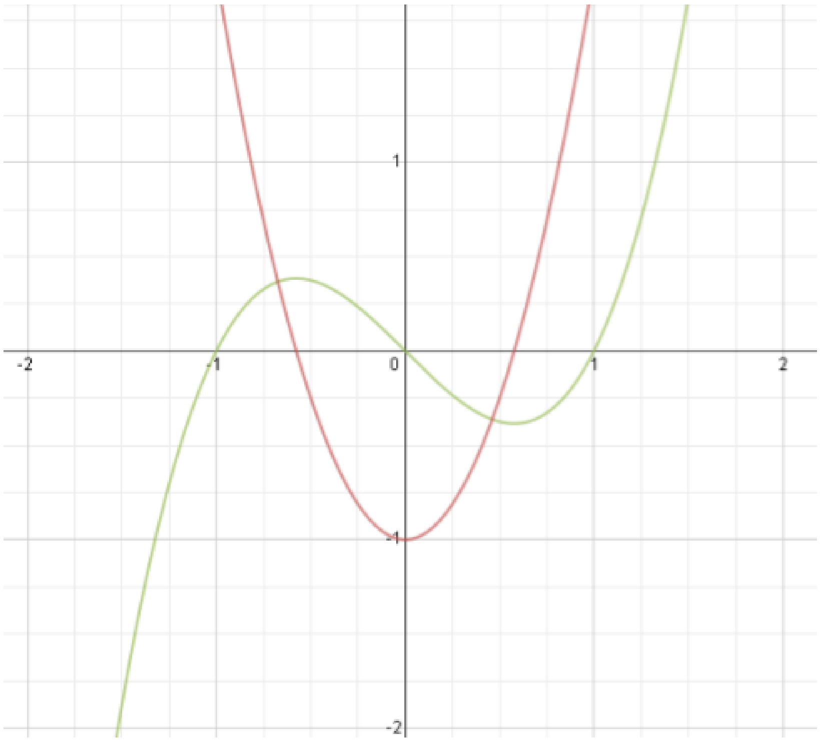
2) $f(x) = x^3 - x$

$$\lim_{h\to 0} \frac{(x+h)^3 - (x+h) - \boxed{x^3 - x}}{h} = \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x} - h - \cancel{x^3} + \cancel{x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 - 1)}{\cancel{h}} = 3x^2 - 1 = f'(x)$$

$f(x)=x^3-x$

$f'(x)=2x^2-1$

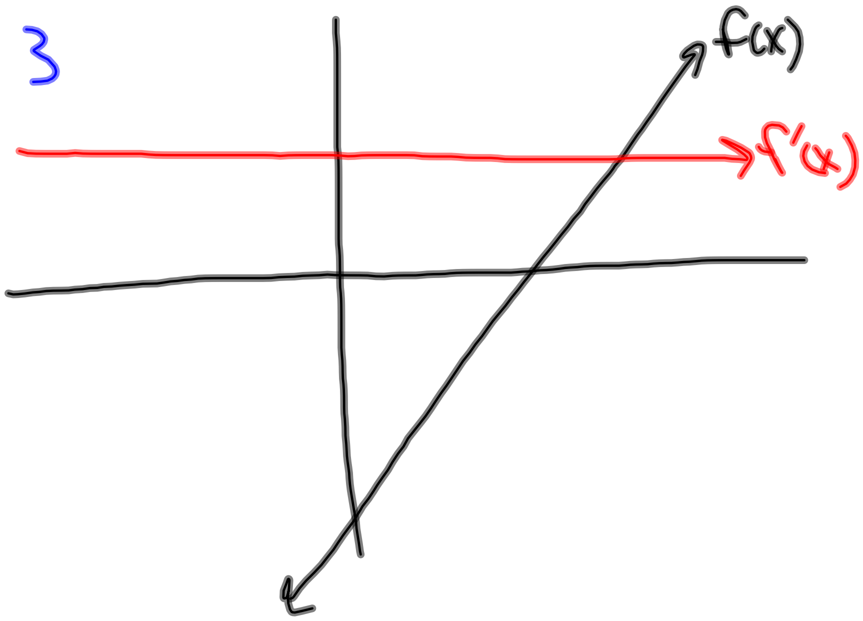


$f(x) = 3x - 6$

$f'(x) = \lim_{h \to 0} \frac{3(x+h) - 6 - (3x-6)}{h} = \lim_{h \to 0} \frac{3x + 3h - 6 - 3x + 6}{h}$

$= \lim_{h \to 0} \frac{3h}{h} = 3$

$f'(x) = 3$



ex 4)

$$f(x)=\sqrt{x}$$

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}$$

$$=\lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

$$f'(9)=\frac{1}{2\sqrt{9}}=\frac{1}{6}$$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$\lim_{x \to 0^+} f'(x) = \infty$

$\lim_{x \to +\infty} f'(x) = 0$

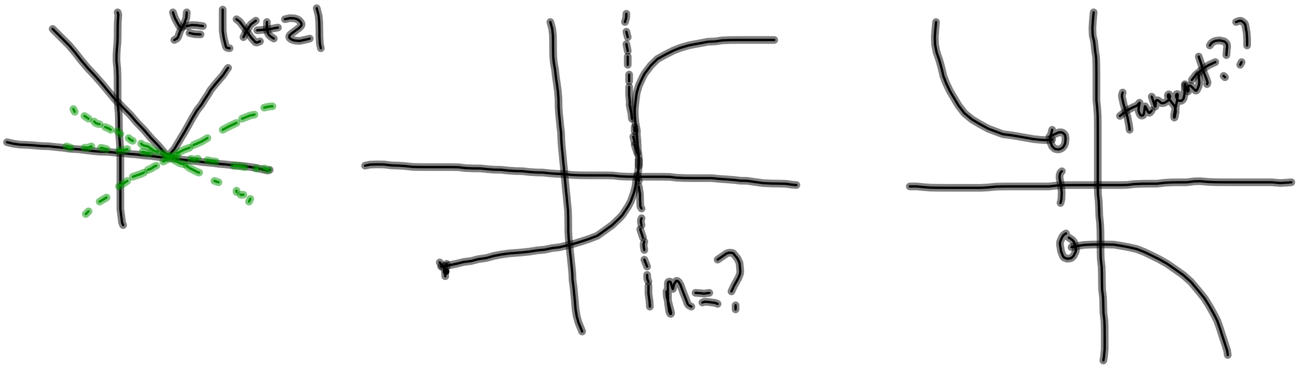
as x gets close to 0, the slope of f(x) becomes large (+). ie, f(x) is getting close to vertical

as x gets large w/o bound the slope of f(x) gets close to zero so the curve is getting close to horizontal

Differentiability

corners,vertical tangents,
points of discontinuity

not differentiable



if a function is differentiable at a point, then it is continuous at that point (the reverse is not necessarily true)

Notation(p184):

$f'(x)$

$$\frac{d\,[f(x)]}{dx} = f'(x)$$

$y = f(x)$

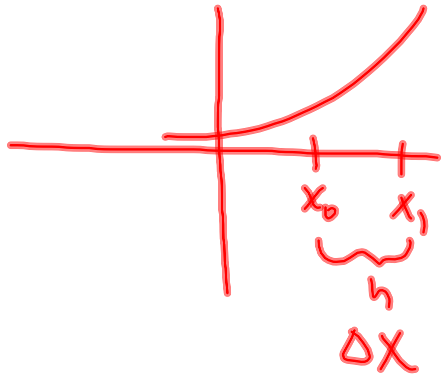
$$\frac{dy}{dx} = f'(x)$$

f'

y' ← more common

Notation(p184):

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



Homework:
pg 175 1,10,11,21
pg 186 3b,7,9,15,23,27,31,39