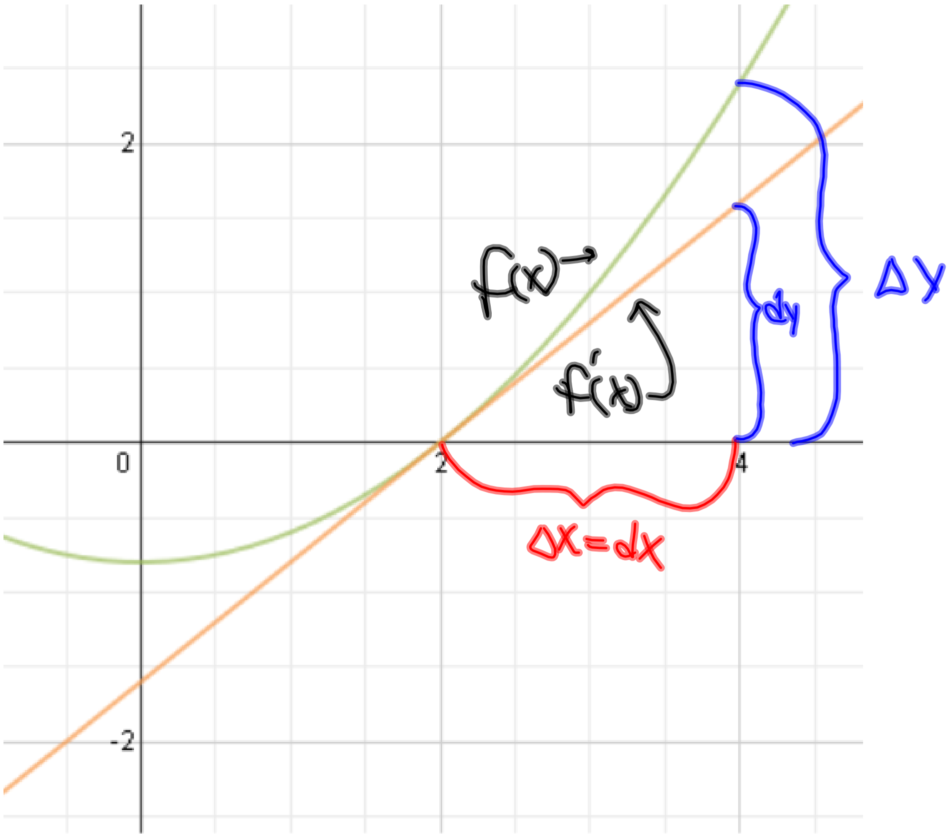
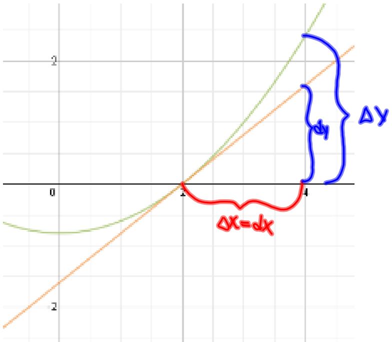
Differentials (sounds impressive, but it is pretty simple and intuitive)



$\frac{\Delta y}{\Delta x}$ = change along curve

$\frac{dy}{dx}$ = change along tangent



define: $dy = f'(x)\,dx$

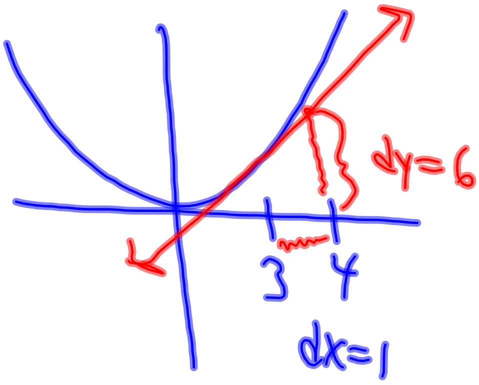
so that we get: $f'(x) = \frac{dy}{dx}$

Δy is NOT the same as dy

Examples 1,2

$y = x^2 \qquad \frac{dy}{dx} = 2x \qquad dy = 2x\,dx$

$@\ x = 3 \qquad dy = 2(3)\,dx$
$= 6\,dx$



ex 2

$y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$dy = \frac{dx}{2\sqrt{x}}$

$x = 4$

$\Delta x = dx = 3$

$x + \Delta x = 7$

$dy = \frac{3}{2\sqrt{4}} = \frac{3}{4}$

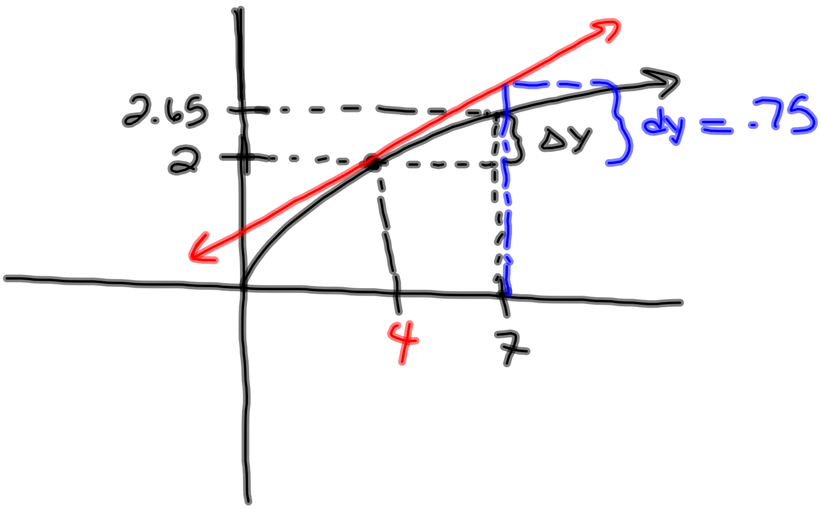
$\Delta y = y_1 - y_0$

$= f(x_1) - f(x_0)$

$= f(7) - f(4)$

$= \sqrt{7} - 2$

$= .65$



2.65
2
Δy
dy = .75
4
7

local linear approximations (ex 3,4)

$$y - f(x_0) = f'(x_0)(x - x_0)$$

tan → $y = f(x_0) + f'(x_0)(x - x_0)$

func → $f(x_0) \approx f(x_0) + f'(x_0)(x - x_0)$

func

tan

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$$

ex3 $\quad f(x) = \sin x \quad x=0$

$\sin x \approx \sin x_0 + (\cos x_0)(x - x_0)$ L.L.A.

@ $x_0 = 0$ $\quad \sin x \approx \sin 0 + (\cos 0)(x - 0)$

$\sin x \approx 0 + (1)(x)$

$\sin x \approx x$

$\frac{180°}{90} = \frac{\pi}{90}$

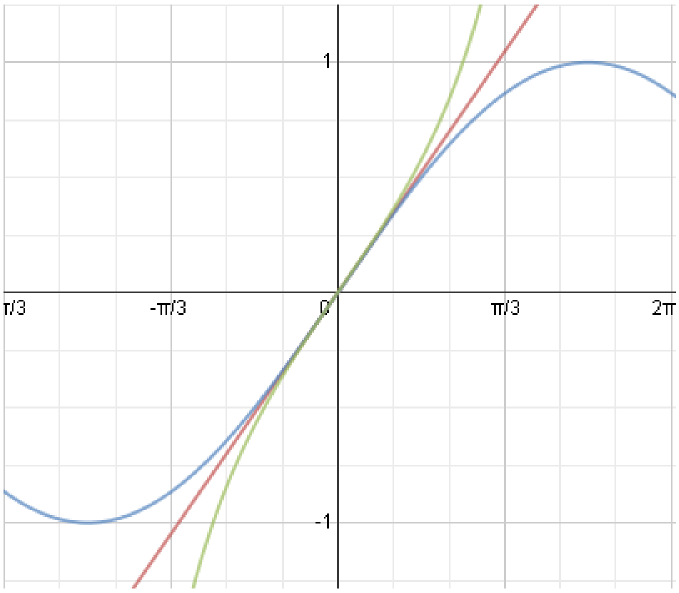
$2 = \frac{\pi}{90}$

$\sin \frac{\pi}{90} \approx \frac{\pi}{90}$

$(\sin 2°)$

$0.03490 \approx 0.03491$

"small angle approximation"
y=sinx, y=tanx, y=x



HW:

Pg 217   2,9,11,13,27,41