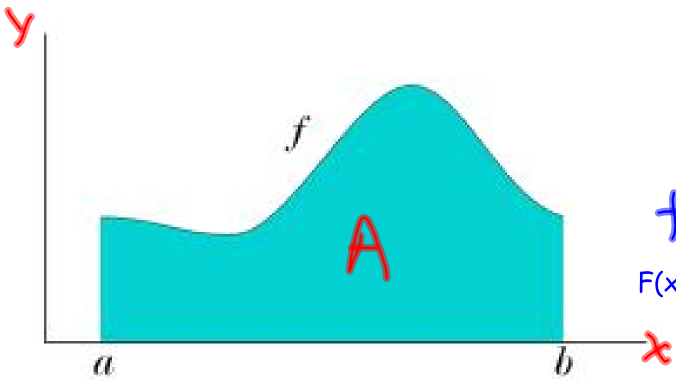
The Fundamental Theorem of Calculus

this sounds important!



$A'(x) = f(x)$
$A(a) = 0$
$A(b) = A$

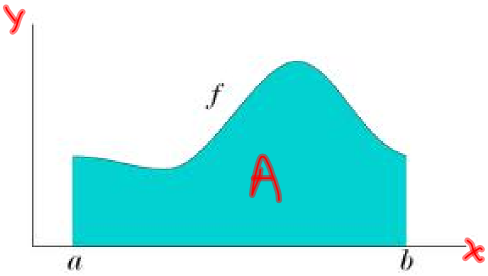
take $F(x) = A(x) + C$

F(x) is one of many antiderivatives of f(x)

$F(b) - F(a) = A$ correct?

(over)

$F(b) - F(a) = A$ correct?



$$F(b) - F(a) = A(b) + \cancel{C} - [\cancel{A(a)}^{0} + \cancel{C}]$$
$$= A(b)$$
$$= A$$

so, we have another expression for the area under the curve, and we write it thusly...

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

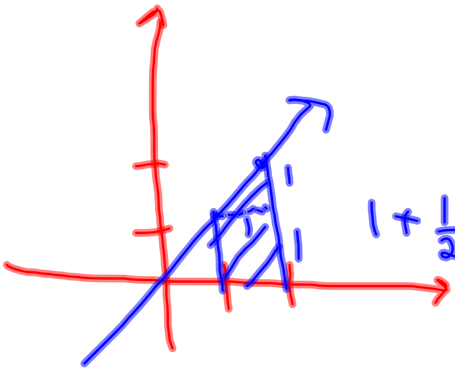
impressive, isn't it?

also written...

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b$$

ex:

$$\int_1^2 x\,dx = \frac{x^2}{2}\Big|_1^2 = \frac{2^2}{2} - \frac{1^2}{2} = 2 - \frac{1}{2} = \frac{3}{2}$$



$1 + \frac{1}{2} = 3/2$

Jul 23-2:55 PM

ex 2)

$$\int_0^3 (9-x^2)\,dx = 9x - \frac{x^3}{3}\Big|_0^3$$

$$= \left(9(3) - \frac{(3)^3}{3}\right) - 0$$

$$= 27 - \frac{27}{3}$$

$$= 18$$

Jul 23-2:59 PM

ex3] $\int_0^{\pi} \cos x\,dx = \sin x \Big|_0^{\pi} = \sin\pi - \sin 0$
$= 0 - 0 = 0$

ex4] $\int_1^9 \sqrt{x}\,dx = \frac{2x^{3/2}}{3} \Big|_1^9 = \frac{2}{3}(9)^{3/2} - \frac{2}{3}\cdot 1 = 18 - \frac{2}{3}$
$= \frac{52}{3}$

ex5] $\int_0^{\ln 3} 5e^x\,dx = 5e^x \Big|_0^{\ln 3} = 5e^{\ln 3} - 5e^0 = 15 - 5 = 10$

Jul 24-9:18 AM

425 #4) $f(x) = x^4$ $[-1, 1]$ find area under curve.

"signed area"

$$\int_{-1}^{1} x^4 dx = \frac{x^5}{5}\Big|_{-1}^{1}$$

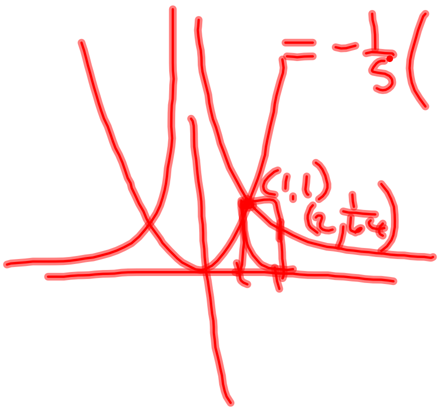
$$= \frac{1}{5} - \left(\frac{-1}{5}\right) = \frac{2}{5}$$

Jul 24-9:29 AM

#12

$$\int_1^2 \frac{1}{x^6}\,dx = \int_1^2 x^{-6}\,dx = \frac{x^{-5}}{-5}\Big|_1^2$$

$$= -\frac{1}{5}\left(\frac{1}{2^5} - 1\right) = -\frac{1}{5}\left(-\frac{31}{32}\right) = \frac{31}{160}$$

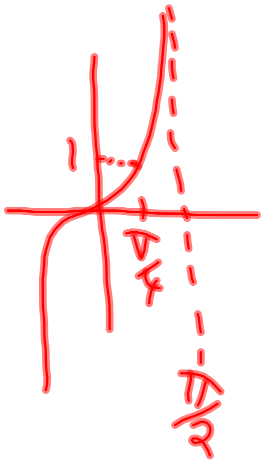


16) $\int_0^{\pi/4} \sec^2\theta \, d\theta$

$= \tan\theta \Big|_0^{\pi/4} = 1 - 0 = 1$

$\frac{d(\tan\theta)}{d\theta} = \sec^2\theta$

$y = \tan\theta$



24) $\int_1^2 (x^{-1} + \sqrt{2}e^x - \csc x \cot x)\,dx$

$\frac{d(\csc x)}{dx} = -\csc x \cot x$

$\ln|x| + \sqrt{2}e^x + \csc x \Big|_1^2$

$\ln 2 + \sqrt{2}e^2 + \csc(2) - (\sqrt{2}e + \csc(1))$

HW:

page 425 3,5,7,11,15,19,23,27b

Jul 23-2:59 PM

speed,velocity,acceleration

$$\int v(t)\,dt = s(t)$$

$$\int a(t)\,dt = v(t)$$

Jul 24-7:42 AM

Uniformly accelerated motion

$a = \text{constant}$

$$s(t) = s_0 + v_0 t + \frac{1}{2} a t^2$$

s_0 - initial position
v_0 - initial velocity
a - constant

$$v(t) = v_0 + at$$

Jul 24-10:21 AM

ex2 p 429

$a = 0.032\ m/s^2$

$\frac{\Delta v}{t}$ $\frac{m/s}{s}$

$\frac{m}{s} \cdot \frac{1}{s} = \frac{m}{s^2}$

$v(0) = v_0 = 10{,}000\ m/s$

$s(0) = s_0 = 0$ (you pick!)

$s(t) = s_0 + v_0 t + \frac{1}{2} a t^2$

1 hour = 3600 s

$s(3600) = 0 + 10{,}000(3600) + \frac{1}{2}(0.032)(3600)^2$

$= 36{,}207{,}400\ m$

$v(3600) = v_0 + at = 10{,}000 + 0.032(3600) = 10{,}115\ m/s$

"free fall" $a = g$

g = accel due to gravity

$= 9.8\ m/s^2$

$= 32\ ft/s^2$

$s(t) = s_0 + v_0 t + \frac{1}{2} g t^2$

$v(t) = v_0 + g t$

Jul 24-10:32 AM

e5 pg 431

$s(t) = s_0 + v_0 t + \frac{1}{2} g t^2$

$v(t) = v_0 + gt$

"from rest" $V_0 = V(0) = 0$ ft/s

$S_0 = 1250$ ft (ground = 0 ft)

$s(t) = s_0 + v_0 t + \frac{1}{2} g t^2$

$0 = 1250 + 0 + \frac{1}{2}(-32)t^2$

$-1250 = -16t^2$

$t = \pm 8.8$ s

$t = 8.8$ s

$V(8.8) = 0 + (-32)(8.8)$

$= -282.8 \frac{ft}{sec}$

ex6 pg 433 $V(t) = t^2 - 2t$ $[0, 3]$

$$s(t) = \int_0^3 (t^2 - 2t)\,dt = \frac{t^3}{3} - t^2 \Big|_0^3$$

$$= \frac{27}{3} - 9 = 0\text{ m}$$

$t^2 - 2t = 0$

$t(t-2) = 0$

$\cancel{t=0}$ $t = 2$

Vel is (-) on $[0, 2)$

Vel is (+) on $(2, 3]$

$$-\int_0^2 (t^2 - 2t)\,dt + \int_2^3 (t^2 - 2t)\,dt$$

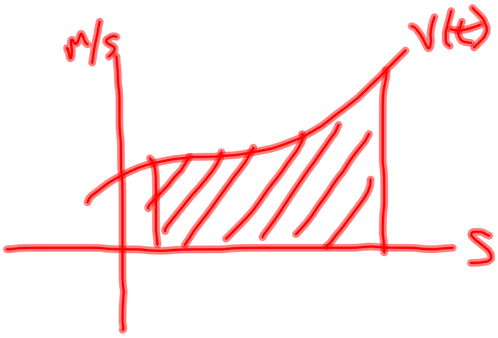
$$-\int_0^2 (t^2-2t)\,dt + \int_2^3 (t^2-2t)\,dt$$

$$-\left(\frac{t^3}{3}-t^2\right)\Big|_0^2 + \left(\frac{t^3}{3}-t^2\right)\Big|_2^3$$

$$-\left(\frac{8}{3}-4\right) + \cancelto{0}{\left(\frac{27}{3}-9\right)} - \left(\frac{8}{3}-4\right)$$

$$-\frac{16}{3}+8 = \frac{8}{3}\text{ m}$$

Jul 24-10:48 AM

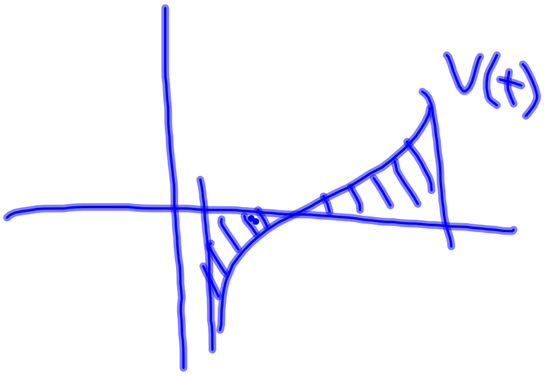


units $A = \frac{m}{s} \cdot s = m$

area under $v(t) = \int v(t) = s(t)$

area under vel = dist

(when $v(t) > 0$)



area under vel curve
= displacement

Jul 24-10:52 AM

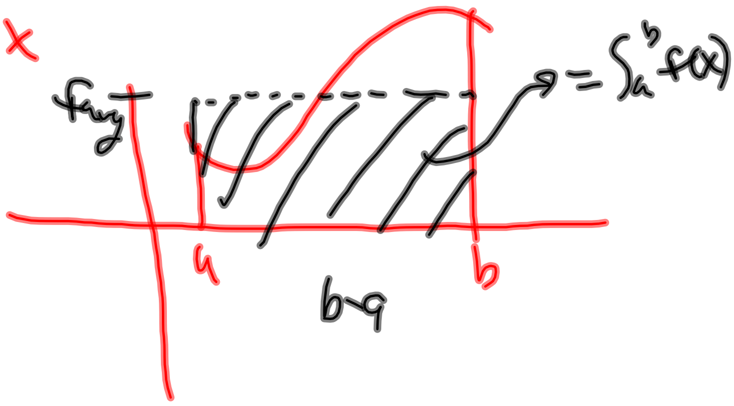
Average value

def 7.7.5 pg 436
fig 7.7.10
ex 10

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

on $[a,b]$

$$(f_{avg})(b-a) = \int_a^b f(x)\,dx$$



$f(x) = \sqrt{x} \quad [1, 4]$

$$f_{avg} = \frac{1}{4-1}\int_1^4 \sqrt{x}\,dx = \frac{1}{3}\left(\frac{2x^{3/2}}{3}\Bigg|_1^4\right)$$

$$= \frac{2}{9}\left(4^{3/2} - 1^{3/2}\right)$$

$$= \frac{2}{9}(8-1) = \frac{14}{9}$$

Jul 24-11:01 AM

HW:

pg 437 1a,3,5,7,13a,35,53