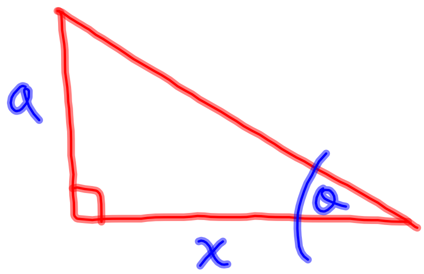
Integrating with Trigonometric Substitutions

$\sqrt{a^2-x^2}$ $\quad\sqrt{x^2-a^2}$ $\quad\sqrt{x^2+a^2}$



$$\sin\theta = \frac{a}{H} \qquad \sin^2\theta + \cos^2\theta = 1$$

$$\cos\theta = \frac{x}{H}$$

$$H = \sqrt{a^2+x^2}$$

ex 1 $\int \frac{dx}{x^2\sqrt{4-x^2}}$

let $x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$x^2 = 4\sin^2\theta$

$\sqrt{4-x^2}$

$\sqrt{4-4\sin^2\theta}$

$\sqrt{4}\sqrt{1-\sin^2\theta}$

$\sin^2\theta+\cos^2\theta=1$

$2\sqrt{\cos^2\theta}$

$2\cos\theta$

$\int \frac{2\cos\theta\, d\theta}{4\sin^2\theta\sqrt{4-4\sin^2\theta}}$

$\int \frac{\cancel{2\cos\theta}\, d\theta}{4\sin^2\theta(\cancel{2\cos\theta})}$

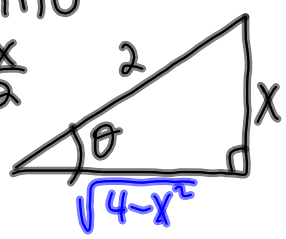
$\frac{1}{4}\int \csc^2\theta\, d\theta = -\frac{1}{4}\cot\theta + C$

$$-\frac{1}{4}\cot\theta + C$$

$$-\frac{1}{4}\frac{\sqrt{4-x^2}}{x} + C$$

$x = 2\sin\theta$

$\sin\theta = \frac{x}{2}$



$2^2 = x^2 + b^2$

$2^2 - x^2 = b^2$

$b = \sqrt{4-x^2}$

ex2

$$\int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}}$$

$$= -\frac{1}{4}\frac{\sqrt{4-x^2}}{x}\Big|_{x=1}^{\sqrt{2}} = -\frac{1}{4}(1) + \frac{1}{4}\sqrt{3} = \frac{\sqrt{3}}{4} - \frac{1}{4}$$

$$= -\frac{1}{4}\cot\theta\Big|_{\theta=?}^{?}$$

$x = 2\sin\theta$

$\sin\theta = \frac{x}{2}$

$\theta = \sin^{-1}\left(\frac{x}{2}\right)$

$$= -\frac{1}{4}\cot\theta\Big|_{\pi/6}^{\pi/4} =$$

$x=1 \Rightarrow \theta = \frac{\pi}{6}$

$x=\sqrt{2} \Rightarrow \theta = \frac{\pi}{4}$

ex 5 $\displaystyle\int \frac{\sqrt{x^2-25}}{x}\,dx$

let $x = 5\sec\theta$

$dx = 5\sec\theta\tan\theta\,d\theta$

$$\int \frac{\sqrt{25\sec^2\theta-25}}{\cancel{5\sec\theta}}\,\cancel{5\sec\theta}\tan\theta\,d\theta$$

$$\int 5\sqrt{\sec^2\theta-1}\,\tan\theta\,d\theta$$

$$\int 5\sqrt{\tan^2\theta}\,\tan\theta\,d\theta$$

$$5\int \tan^2\theta\,d\theta$$

$$\frac{s^2+c^2=1}{c^2\quad c^2\quad c^2}$$

$\tan^2+1=\sec^2$

$$5\int \tan^2\theta \, d\theta$$

$$= 5\int (\sec^2\theta - 1)\, d\theta$$

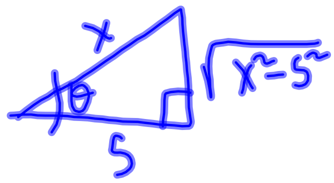
$$= 5(\tan\theta - \theta) + C$$

$$= 5\left(\frac{\sqrt{x^2-5^2}}{5} - \sec^{-1}\left(\frac{x}{5}\right)\right) + C$$

$$= \sqrt{x^2-5} - \frac{1}{5}\sec^{-1}\left(\frac{x}{5}\right) + C$$

$x = 5\sec\theta$

$\sec\theta = \frac{x}{5}$



$x^2 = b^2 + 5^2$

ex 7 $\int \frac{dx}{\sqrt{5-4x-2x^2}}$

$-2x^2-4x+5$

$-2\left(x^2+2x-\frac{5}{2}\right)$

$-2\left(x^2+2x+1\right)+5+2$

$-2(x+1)^2+7$

$\int \frac{dx}{\sqrt{7+-2(x+1)^2}}$

let $u=x+1$ $du=dx$

$\int \frac{du}{\sqrt{7-2u^2}} = \int \frac{du}{\sqrt{\frac{7}{2}-u^2}} =$

$\sqrt{a^2-u^2}$

$a=\sqrt{\frac{7}{2}}$

$$\int \frac{du}{\sqrt{\left(\sqrt{\frac{7}{2}}\right)^2 - u^2}} = \sin^{-1}\left(\frac{u}{\sqrt{7/2}}\right) + C$$

$$= \sin^{-1}\left(\frac{x+1}{\sqrt{7/2}}\right) + C$$

HW:

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